$$(1\div\tfrac{1}{4}a^2)\int_0^{\frac{1}{4}a^2}\beta db=\frac{4}{a^2}\int_0^{\frac{1}{4}a^2}(\tfrac{1}{2}a+\surd(\tfrac{1}{4}a^2-b)db=\frac{4}{a^2}[\tfrac{1}{2}ab]_0^{\frac{1}{4}a^2}+\frac{4}{a^2}[\tfrac{1}{12}\surd(a^2-4b)^3]_0^{\frac{1}{4}a^2}$$

$$-\frac{a}{3}=\frac{a}{2}-\frac{a}{3}=\frac{1}{6}a.$$

The mean value of the larger root is, therefore, $\frac{5}{6}a$.

Also solved in a similar manner by *Professors Matz, Zerr, and Draughon.*

14. Proposed by CHARLES E. MYERS, Canton, Ohio.

$\frac{1}{3}$ of all the melons in a patch are not ripe, and $\frac{1}{4}$ of all the melons in the same patch are rotten, the remainder being good. If a man enters the patch on a dark night and takes a melon at random, what is the probability that he will get a good one?

Solution by H. W. DRAUGHON, Olio, Mississippi, and G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let $12n$=the whole number of melons in the patch. Then $4n$ are not ripe and $3n$ are rotten. The $3n$ rotten melons may be included in the $4n$ not ripe melons in which case there would be $8n$ good melons, or the $3n$ rotten may not be included in the $4n$ not ripe melons in which case there would be $12n-(3n+4n)=5n$ good melons.

$\therefore$ there cannot be less than $5n$ nor more than $8n$ good melons.

$\therefore$ the chance of a good one$=\frac{1}{2}\left(\dfrac{5n+8n}{12n}\right)=\frac{13}{24}$.

The chance of a not ripe one$=\frac{1}{2}\left(\dfrac{n+4n}{12n}\right)=\frac{5}{24}$.

The chance of a rotten one$=\frac{1}{2}\left(\dfrac{0+3n}{12n}\right)=\frac{1}{8}$.

The chance of a not ripe and rotten one$=\frac{1}{2}\left(\dfrac{0+3n}{12n}\right)=\frac{1}{8}$.

$\therefore$ $\frac{13}{24}+\frac{5}{24}+\frac{1}{8}+\frac{1}{8}=1$ as it should be.

Solutions o this problem were received from *P. S. Berg, F. P. Matz, J. M. Colaw*.

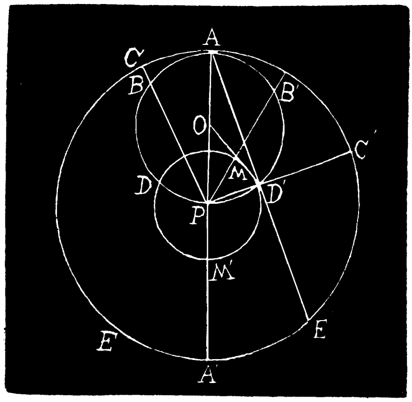
15. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Todhunter proposes: "From a point in the circumference of a circular field a projectile is thrown at random with a given velocity, which is such that the diameter of the field is equal to the greatest range of the projectile: prove the chance of its falling within the field, is $C=2^{-1}-2\pi^{-1}(\sqrt{2}-1)$, $=.236+$." Is this result perfectly correct as to fact?

First Solution by the PROPOSER.

Let P be the point from which the projectile is thrown, $AP=2a$, and $\angle APB=\theta$. Now, if ϕ=the angle of elevation at which the projectile is thrown, and C'=the chance for any given value of θ; then, evidently, the required chance becomes

$$C=\int_0^{\frac{1}{2}\pi} C'\,d\theta \div \int_0^{\pi} d\theta = \frac{1}{\pi}\int_0^{\frac{1}{2}\pi} C'\,d\theta \ldots (1).$$



Since the range is

$$PB=2a\sin 2\phi=2a\cos\theta,$$

$\sin\phi=\frac{1}{2}[\sqrt{(1+\cos\theta)}+\sqrt{(1-\cos\theta)}],=R_1$, and $\sin\phi=\frac{1}{2}[\sqrt{(1+\cos\theta)}-\sqrt{(1-\cos\theta)}]$, $=R_2$.

For all values of $\sin\phi$ *less* than R_2 and *greater* than R_1, the projectile will fall into the field. The whole number of different directions of projection is proportional to the surface S_1 of the hemisphere center of which is P and radius $\frac{1}{2}PB=a\cos\theta$; and this surface is

$$S_1=2\pi a^2\cos^2\theta \ldots (2).$$

The whole number of different directions of projection producing a range *greater* than PB is proportional to the surface S_2 of the zone included between two horizontal planes at the distances $R_1 a\cos\theta$ and $R_2 a\cos\theta$ from the center of the base of the hemisphere; and this surface is

$$S_2=a\cos\theta\sqrt{(1-\cos\theta)}\times 2\pi a\cos\theta=2\pi a^2\cos^2\theta\sqrt{(1-\cos\theta)} \ldots (3).$$

That is, the whole number of different directions of projection giving a range *less* than PB is proportional to S_1-S_2; and, therefore, we have the chance for any given value of θ

$$C'=\frac{S_1-S_2}{S_1}=1-\frac{S_2}{S_1}=1-\sqrt{(1-\cos\theta)}=1-\sqrt{(2)}\sin\tfrac{1}{2}\theta \ldots (4).$$

$$C=\frac{1}{\pi}\int_0^{\frac{1}{2}\pi} C'\,d\theta=\frac{1}{\pi}\int_0^{\frac{1}{2}\pi}[1-\sqrt{(2)}\sin\tfrac{1}{2}\theta]d\theta=\frac{1}{2}-2\left(\frac{\sqrt{2}-1}{\pi}\right)$$

$=2^{-1}-2\pi^{-1}(\sqrt{2}-1),=.236+$, which is Todhunter's result.

Second Solution.

Obviously the number of *favorable* chances is represented by the area of the circle $ABDPD'B'A-O$, and the total number of chances is represented by the area of the circle $ACEA'E'C'A-P$. Therefore, the required chance is $C=\frac{1}{4},=.25$.

Third Solution.

For any range PM the number of *favorable* chances is represented by the area $\mathbf{A}'$ of the double segment $DMD'P$, and the total number of chances is represented by the area of the circle $DM'D'M-P$. Let $\angle PAD'=\omega$; then $\angle POD'=2\omega$, $\angle MPD'=(90-\omega)$, $PM=PD'=2a\sin\omega$, and $\mathbf{A}'=2$(Sector MPD'+Sector POD'−Triangle $OD'P$)$=2a^2[\pi\sin^2\omega$ $-(\omega+\sin\omega\cos\omega-2\omega\cos^2\omega)]$. Hence the required chance becomes

$$C=2a^2\int_0^{\frac{1}{2}\pi}\mathbf{A}'d\omega+4\pi a^2\int_0^{\frac{1}{2}\pi}\sin^2\omega d\omega=\frac{1}{2}-\frac{2}{\pi^2},=.297+.$$

Fourth Solution.

From A draw at random the chord AE', put $\angle A'AE'=\Psi$; then $AD'=2a\cos\Psi=r_1$, and $AE'=4a\cos\Psi=r_2$. For any value of Ψ the number of *favorable* chances is represented by the *sectoral* surface PAD', and the total number of chances by the sectoral surface $A'AE'$. The chance in consideration, therefore, becomes

$$C=2\int_0^{\frac{1}{2}\pi}\int_0^{r_1} d\Psi rdr\div 2\int_0^{\frac{1}{2}\pi}\int_0^{r_2} d\Psi rdr=\frac{1}{4},=.25.$$

Fifth Solution.

Put $\angle PAD'=\omega$, then $\angle APD'=(\frac{1}{2}\pi-\omega)$. Therefore, for any range PD' the projectiles falling on the circular arc DMD' are within the field. Consequently the required chance becomes

$$C=4a\int_0^{\frac{1}{2}\pi}(\tfrac{1}{2}\pi-\omega)\sin\omega d\omega\div 4\pi a\int_0^{\frac{1}{2}\pi}\sin\omega d\omega=\frac{1}{2}-\frac{1}{\pi},=.182.$$

Sixth Solution.

The number of *favorable* chances is proportional to $2\angle MPD'$ $=2(\frac{1}{2}\pi-\Psi)$, and the total number of chances is proportional to $2(\pi)$. Hence the required chance becomes

$$C=2\int_0^{\frac{1}{2}\pi}(\tfrac{1}{2}\pi-\Psi)d\Psi\div 2\pi\int_0^{\frac{1}{2}\pi}d\Psi=\tfrac{1}{4},=.25.$$

NOTE—Since the projectiles are *thrown* at random, they should *fall* at random; and, therefore, the required chance should be $C=\frac{1}{4},=.25$. To interpret *this* result is apparently easy enough; but to interpret Todhunter's result, or the results $C=.297+$and $C=.182$, is not so easy. In fact, the interpretation of these three results becomes all the more remarkable when we note that their average value $C_A=.238+$, which average value differs but slightly from Todhunter's result.

This problem was also solved by G. B. M. Zerr. J. M. Colaw, and John Dolman, Jr., their result agreeing with that given by Todhunter and the first solution of Professor Matz. Professor Zerr says this result is perfectly true as to mathematical fact. We published all of Professor Matz's solutions for comparison.—EDITOR.

18. Proposed by B. F. FINKEL, A. M., Professor of Mathematics in Kidder Institute, Kidder, Missouri.

What is the average volume common to a cube and a rectangular solid one inch square, the axis of rectangular solid being equal to and coinciding with the diagonal of the cube?

Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let $ABCD$ be a section of the rectangular solid, EFG a section of